The case study of application Hilbert transform in ECG signal processing

Danilo Janković
Faculty of Electrical Engineering
University of Montenegro
Podgorica, Montenegro
danilo.jankovic93@gmail.com

Radovan Stojanović
Faculty of Electrical Engineering
University of Montenegro
Podgorica, Montenegro
stox@ac.me

Abstract— **In this paper, application of Hilbert transform in biomedical signal processing is going to be demostrated using ECG signal. The Hilbert transform is a linear operator that takes a function $u(t)$ of a real variable and transforms it into another function of a real variable H(u)(t). The Hilbert transform is important in mathematics and signal processing, where it is a component of the analytic representation of a real-valued signal $u(t)$. The Hilbert transform was first introduced by German matemathician David Hilbert, to solve a special case of the Riemann–Hilbert problem for analytic functions.**

Keywords-Hilbert transform; Fourrier transform; signal processing; ECG; peak detection

I. INTRODUCTION

A. Definition

The Hilbert transform of given function u can be defined as the convolution of $u(t)$ with the function $h(t) = 1/(\pi t)$. The function $1/(\pi t)$ is known as the Cauchy kernel. The integral defining the convolution does not always converge since $1/t$ is not integrable for $t = 0$. Instead, the Cauchy principal value (denoted here as p.v.) is used for Hilbert transform definition.[1][2] Now, the Hilbert transform of a function (or signal) $u(t)$ can be defined as:

$$H(u)(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{+\infty} \frac{u(\tau)}{t-\tau} d\tau, \quad (1)$$

provided this integral exists as a principal value. Alternatively, by changing variables, the principal value integral can be written explicitly as:[3]

$$H(u)(t) = -\frac{1}{\pi} p.v. \lim_{\varepsilon \to 0} \int_{\varepsilon}^{+\infty} \frac{u(t+\tau) - u(t-\tau)}{\tau} d\tau. \quad (2)$$

If the Hilbert transform is applied twice to a function u, the result is negative u:

$$H\big(H(u)\big)(t) = -u(t), \quad (3)$$

under the condition that the integrals defining both iterations converge in a suitable sense.

Considering an analytic function in the upper half of the complex plane, the relationship between the real part and the imaginary part of the boundary values is shown using Hilbert transform. In other words, if $f(z)$ is analytic in the upper half of the complex plane $\{z\text{: Im}\{z\} > 0\}$ and $u(t) = \text{Re}\ \{f(t + 0\cdot i)\}$, then $\text{Im}\ \{f(t + 0\cdot i)\} = \text{H}(u)(t)$ up to an additive constant, provided this Hilbert transform exists.

B. Relation with Fourier transform

The Hilbert transform is a multiplier operator.[4] The multiplier of H is $\delta_{\text{H}}(\omega) = -i\ \text{sgn}(\omega)$, where sgn is the signum function. Therefore:

$$F\big(H(u)\big)(\omega) = -isgn(\omega)F(u)(\omega), \quad (4)$$

where F denotes the Fourier transform. Since $\text{sgn}(x) = \text{sgn}(2\pi x)$, it follows that this result applies to the three common definitions of F.

Considering Euler's formula:

$$\sigma_H(\omega) \begin{cases} i = e^{+\frac{i\pi}{2}}, & for\ \omega < 0 \\ 0, & for\ \omega = 0 \\ -i = e^{-\frac{i\pi}{2}}, & for\ \omega > 0 \end{cases} \quad (5)$$

$\text{H}(u)(t)$ actually means shifting the phase of the negative frequency components of function $u(t)$ by +90° ($\pi/2$ radians) and, on the other hand, the phase of the positive frequency components by −90°. Also, $i\cdot\text{H}(u)(t)$ has the effect of restoring the positive frequency components while shifting the negative frequency ones an additional +90°, resulting in their negation (i.e., a multiplication by −1).

When the Hilbert transform is applied twice, the phase of the negative and positive frequency components of $u(t)$ are respectively shifted by +180° and −180°, which are equivalent amounts. In this way, the signal is negated, i.e., $\text{H}(\text{H}(u)) = -u$, because:

$$(\delta_H(\omega))^2 = e^{+i\pi} = -1,\ \ for\ \omega \neq 0. \quad (6)$$

Hilbert transform is very useful when it comes to peak finding logic in ECG signals and can be extended to PPG signal, as example for purposes of HRV analysis. [5]

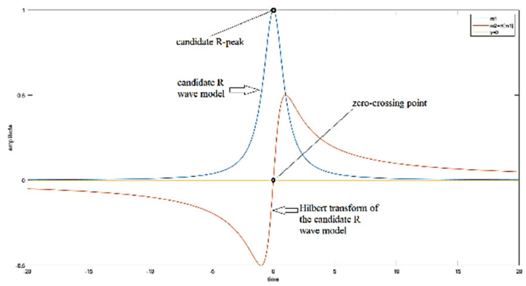

Fig. 1. R-peak finging logic using Hilbert transform

II. METHOD

In order to demonstrate the maxima finding task using the Hilbert transform, we are going to take the even function, $m_1(t) = 1/(1+t^2)$, which is considered as a model of R-wave envelope. The Hilbert transform of such function, $m_2(t)$, is given by $H[m_1(t)] = m_2(t) = t/(1+t^2)$. The graphics of the $m_1(t)$ and $m_2(t)$ functions are presented in Fig. 1. It can be observed that the maximum amplitude value of the envelope function $m_1(t)$ corresponds to the zero-crossing point of the $m_2(t)$.

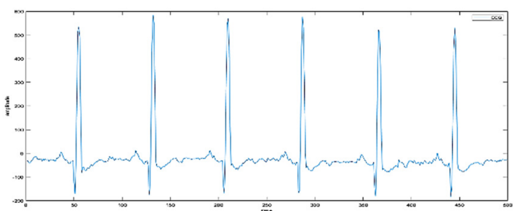

Fig. 2. ECG signal (first 6 pulses)

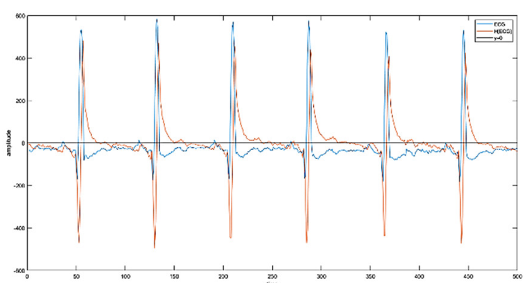

Fig. 3. ECG signal and its Hilbert transform

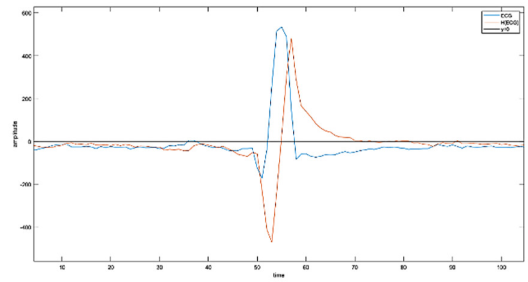

Fig. 4. First ECG pulse and its Hilbert transform

III. DEMONSTRATION

Hilbert transform has wide application in signal processing, especially in biomedical signal processing. Application of Hilbert transform can be shown using ECG signal (Fig. 2). This signal is obtained from MIT-BIH Normal Sinus Rhythm Database from PhysioNet website (record: 16265, signal: ECG1, length: 1 hour) [6].
Now, we are going to apply Hilbert transform on mentioned ECG signal (Fig. 3). Let's now emphasize first ECG pulse and its Hilbert transform. (Fig. 4).

IV. RESULTS

As can be seen from figures above, Hilbert transform modifies ECG in a way that zero crossing points (points where amplitude of a signal crosses x-axis from negative to positive part) in resulting signal are matched with R-peaks of original ECG pulses. So, the identification of R-peaks (highest amplitude value in ECG pulses) is based on detection of a time instance when Hilbert transform of the ECG signal crosses function *y=0*, which is much easier way for peak detection.

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