About Time-Frequency Filters, Challenges and Experiences

Veselin N. Ivanović[1], Srdjan Jovanovski[2]
[1] Department of Electrical Engineering
University of Montenegro
81000 Podgorica, Montenegro
very@ac.me
[2] Faculty of Information Technology
Mediterranean University
81000 Podgorica, Montenegro
srdjaj@t-com.me

***Abstract*— Optimal (Wiener) filter based on the results of time-frequency analysis in estimation of instantaneous frequency (IF) of the estimated nonstationary signals and on the correspondence of the estimated IF to the filter's region of support provide high quality estimation of the linear frequency modulated (FM) noisy signals. Signal adaptive multiple-clock-cycle and completely pipelined hardware designs of this filter, with the optimized time and hardware requirements and in the one-dimensional and two-dimensional form, have also been developed. However, the IF estimation-based filtering solutions do not provide satisfactory results in estimation of the non-liner FM signals. Development of the modified IF estimation algorithm suitable to provide high quality estimation of the non-linear FM signals represents our topic in progress.**

Keywords*— *Estimation, Instantaneous frequency, Non-linear FM signals, Time-varying filter.

I. Introduction

Signals usually spread across a range of frequencies whose width grows with nonstationarity of their characteristics. Therefore, signals with stationary, or almost stationary, characteristics can be efficiently analyzed by using conventional time- or frequency-invariant approaches. However, efficient processing of nonstationary one-dimensional (1D) and two-dimensional signals, including their filtering, requires a time-varying, [1-2], or a space-varying approach, [3], respectively. Time-varying approach can be defined by using the common time-frequency (TF) domain tools based on TF distributions (TFDs), whereas space-varying approach can be defined by using the space/spatial-frequency (S/SF) domain tools based on the S/SF distributions. These solutions have usually been referred to as the TF and S/SF filters, respectively.

TF filters can be designed explicitly or implicitly. TF filters, whose regions of support (FRSs) are the best approximations of the transfer functions of the distributions used in their definitions, have explicit design. TF filters having implicit design are based on the calculation of a linear TF transformation of the input 1D signal, on the TFD-based FRS estimation, and on the output signal estimation from the calculated linear transformation multiplied by the estimated FRS. The implicitly designed filters using the FRS estimation based on linear TFDs involve only linear processing steps, as well as explicitly designed filters, so these solutions result in linear TF filters. The implicitly designed filters using the FRS estimation based on quadratic TFDs result in nonlinear TF filtering solutions.

Classical filtering solutions belong to the linear filters, since they have been designed either explicitly based on the Richaczek distribution and the Wigner distribution (WD) (Zadeh and Weyl filter, [1], respectively), or implicitly based on the linear short-time Fourier transform (STFT) and Gabor transform (STFT and Gabor filters, [1], respectively). However, these solutions suffer from the drawbacks introduced by the TFDs used in their definitions. The Zadeh filter cannot be used for nonstationary signals, the STFT and Gabor filters have limited resolution, whereas the Weyl filter is essentially restricted to halfband signals, [1]. To suppress the noted flaws and to extend the limits of classical solutions, their extended versions (the multiwindow STFT filter, the multiwindow Gabor filer, as well as the approximate halfband Weyl filter) have been defined, [1]. However, the extended versions increase calculation complexity of the classical solutions. Nonlinear filtering solutions, based on the WD or the smoothed WD, [1], improve resolution and selectivity, but at the expense of their complexity. The projection filter, [1], has extremely high selectivity, but also significantly higher complexity in comparison to the other solutions.

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Being numerically quite complex, TF filters require significant time for calculation. Therefore, they are usually unsuitable for real-time analysis and thus their application is severely restricted in practice. Hardware implementations, when possible, can overcome these problems. However, online algorithms for single-clock-cycle implementation (SCI) of linear TF filters from [1], the existing SCI design of the nonlinear TF filter from [4], and possible implementations of nonlinear TF filters from [2], require repeating of basic calculation elements if they need to be used more than once. Therefore, these implementation schemes can be so complex that often cannot be implemented. Besides, complexity of these systems depends on the filtered signal duration, so they are capable of performing estimation of signals with the predefined duration only. To overcome flaws of the mentioned TF filters' designs, the multiple-clock-cycle, but also signal adaptive, [5], as well as the completely pipelined, [6], implementations of the IF estimation-based optimal (Wiener) TF filter have recently been developed. These solutions simultaneously optimize complexity and the execution time of the developed filter, making it suitable for real-time and on-a-chip implementation.

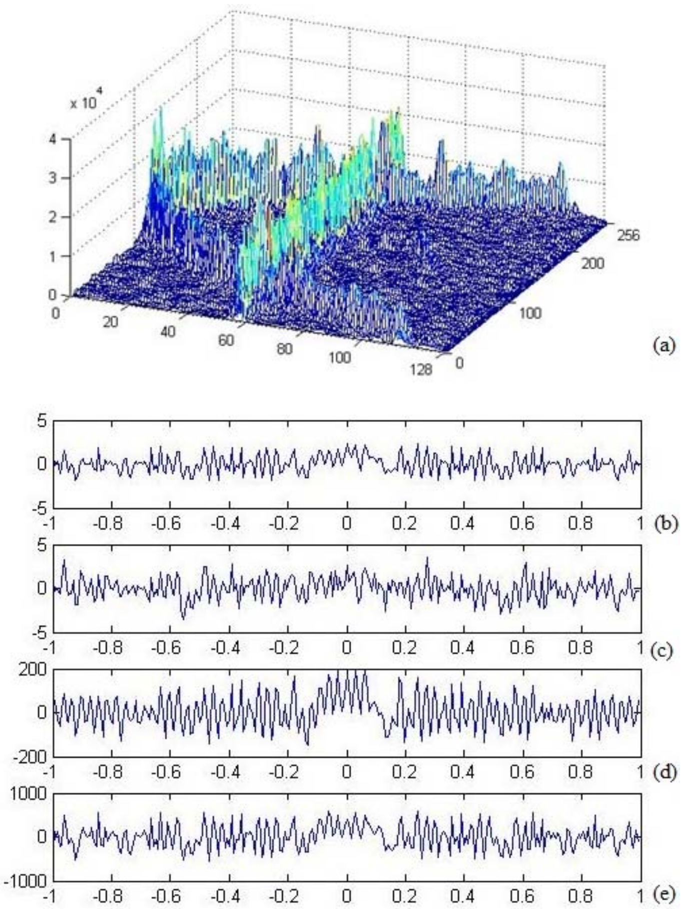

Fig. 1 (a) *CTFWD of the multicomponent noisy signal* (3), (b) *Original multicomponent signal* (3), (c) *Noisy signal* (3), (d) *The estimated signal obtained by using the state-of-the-art IF estimation algorithms from* [2, 5], (e) *The estimated signal obtained by using the modified IF estimation algorithm.*

II. Problem Formulation

Optimal filtering of nonstationary 1D signals, related to the Wigner distribution framework, [1-2], and used to overcome distortion of the estimated FM signals has been defined and can be written, in the frequency domain, [2, 5], by:

$$(Hx)(n) = \sum_{k=-N/2+1}^{N/2} L_H(n,k) STFT_x(n,k) \qquad (1)$$

where $L_H(n,k)$ is the FRS (the Weyl symbol of the filter's impulse response), [1-2], $STFT_x(n,k)=DFT_m[w(m)x(n+m)]$ is the STFT of the noisy q-component signal,

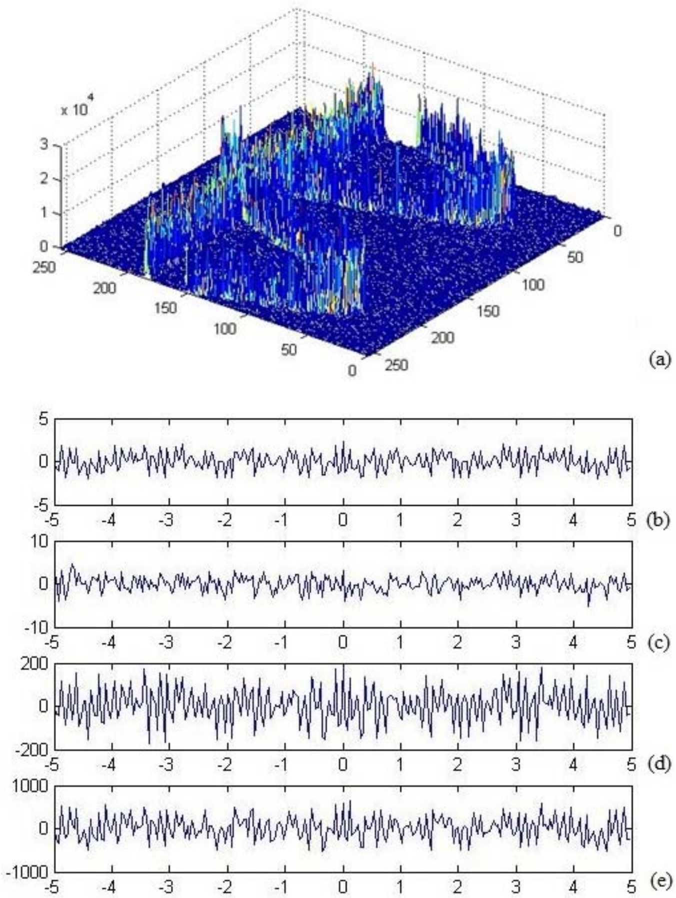

Fig. 2 (a) *CTFWD of the multicomponent noisy signal* (4), (b) *Original multicomponent signal* (4), (c) *Noisy signal* (4), (d) *The estimated signal obtained by using the state-of-the-art IF estimation algorithms from* [2, 5], (e) *The estimated signal obtained by using the modified IF estimation algorithm.*

$$x(n) = \mathrm{sum}_{i=1,..,q}(f_i(n)) + \varepsilon(n),$$

DTF_m[ ] is the operator of the discrete Fourier transform in m, $w(m)$ is a real-valued STFT lag window, and N is the windowed signal duration.

Following procedure for the stationary Wiener filter design in the case of signal not correlated with the additive noise $\varepsilon(n)$, of FM signals $f_i(n)$, $i=1,\ldots,q$, highly concentrated in the TF plane around their IFs, and of a widely spread white noise, the FRS $L_H(n,k)$ of the optimal (Wiener) nonstationary filter corresponds to the combination of IFs of signals $f_i(n)$, [2, 5]. Then, the optimal filtering problem of nonstationary FM signals can be reduced to the IF estimation in a noisy environment. This solution applied to (1) results in a high quality filtering of nonstationary FM signals highly concentrated in the TF plane and exposed to the widely spread white noise, [2, 5]. In the practically most important case of a single realization of noisy signals and the TF analysis framework, the IF estimation is performed by determining frequency points k_i, $i=1,\ldots,q$, in which TF representation of noisy signal has local maximum,

$$IF_i(n) = \arg[\max_{k \in Q_{k_i}} TFD_x(n,k)] \tag{2}$$

where Q_{ki} is the basic frequency region in TF plane around $f_i(n)$, the IF of which is $IF_i(n)$.

Among all the TFDs, the recently defined cross-terms-free Wigner distribution (CTFWD), [5, 7], optimizes the IF estimation characteristics, [8]. In detail, it retains the optimal IF estimation characteristics of the Wigner distribution in the highly nonstationary mono-component signals case, [8]. However, it also produces the optimal IF estimation characteristics in the case of multicomponent signals whose components do not overlap in frequencies. In that case, the IF estimation characteristics of the CTFWD, obtained for each signal's component separately, remain the same as for the case when only that particular signal's component exists, [9]. This qualifies the CTFWD as an optimal base for an IF estimation-based TF filter (1)-(2) development that has be performed in [4-6].

Following characteristics of the classic Wigner distribution, the CTFWD produces optimal TF representation of the linear FM signals providing their highest quality estimation based on the IF estimation related filtering (1)-(2), [2, 5-6]. However, there is no a TF distribution which produces optimal representation of the non-linear FM signals. Besides, in a particular time instant and due to the frequency discretization, the non-linear FM signals can (and usually) occupy certain range of frequencies. Further, under the additive noise influence and depending on the selected Q_{ki} width, each frequency from the considered range can mask more or less the adjacent frequencies from same range of frequencies, disabling their estimation based on the definition (2) and, therefore, disabling efficient estimation of non-linear FM signals.

Taking into account the noted principles, the problem of the non-linear FM signals estimation can be efficiently solved. To this end, the IF estimation algorithm, based on the definition (2) and proposed in [5], should be modified in such a way to recognize the particular time instants in which the considered signal occupies the certain ranges of frequencies and to enable the IF estimation in all frequencies from the considered ranges. Result of the research activities in improvement of the IF estimation algorithm that provides estimation of the non-linear FM signals and the comparisons of the modified algorithm with the state-of-the-art IF estimation algorithms from [2, 5] are presented in examples given in the sequel.

III. Examples

The modified IF estimation algorithm, which principles are considered in the previous section, is verified through the estimation of the following multicomponent test signals:

$$f_1(nT) = e^{j300\pi(nT)^3} + e^{j400(nT)} + \\ + e^{-30(t-1/8)^2}\cos(625(nT+29)^2), \tag{3}$$

$$f_2(nT) = e^{j700\pi\sin(nT)} + e^{j400(nT)} + \\ + e^{-30(t-1/8)^2}\cos(625(nT+29)^2). \tag{4}$$

Each of the observed signals consists of the quite non-linear FM component combined by the linear FM component and the chirp component, Figs. 1(a), 2(a). These signals are considered within the ranges $-1 \le nT \le 1$ and $-5 \le nT \le 5$, respectively, and are masked by the high white noises such that $SNRin_1=$ $=10\log(P_{f1}/P_{\varepsilon1})=0.2086$[dB] and $SNRin_2=10\log(P_{f2}/P_{\varepsilon2})=$ $=0.2411$[dB], respectively. Within the simulation, the Hanning STFT lag window $w(m)$, width of $T_w=0.25$, is applied, as well as $N=256$, and $T=T_w/N$.

Filtering results obtained within the estimation of 3-component signals (3)-(4) are given in Fig. 1(b)-(e) and in Fig.2(b)-(e), respectively. Filtering efficiency achieved by using the modified IF estimation algorithm can easily be noticed by comparing Figs. 1(b) and 1(e), i.e. by comparing Figs. 2(b) and 2(e). As well, the improvement achieved by using the modified IF estimation algorithm can be recognized from Figs. 1(d) and 1(e), i.e. from Figs. 2(d) and 2(e).

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